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A NOTE ON SOME MODELS OF INTUITIONISTIC FUZZY SETS IN REAL LIFE SITUATIONS

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Abstract: After the introduction of intuitionistic fuzzy sets (IFSs), many researchers have confirmed the resourcefulness of IFSs in decision making problems. In this paper, we gave a lucid and comprehensive note on some selected models of IFSs in real life situations such as in diagnostic medicine, career determination, and pattern recognition using normalized Hamming distance measure.

Keywords: career determination, diagnostic medicine, intuitionistic fuzzy sets, pattern recognition.

INTRODUCTION

In [1, 2], intuitionistic fuzzy set (IFS) was introduced as an extension of fuzzy set earlier proposed in [30]. Many researchers have confirmed the resourcefulness of IFSs in decision making problems due to its significance in tackling vagueness or the representation of imperfect knowledge. There are volumes of literature involving the fundamentals and theory of IFSs in [3-13, 15, 18, 19, 21, 31]. Most of the applications of IFSs are carried out using distance measures approach.Many distance measures between intuitionistic fuzzy sets have been proposed and researched in recent years in [8, 23, 24, 27, 28].Distance measure between intuitionistic fuzzy sets is an important concept in fuzzy mathematics because of its wide applications in real world, such as in pattern recognition, machine learning, medical diagnosis, electoral system, career determination, market prediction, and so on as in [8, 14, 16, 17, 20, 22, 25, 26, 29].

In this research article, we present a comprehensive note on some applications of intuitionistic fuzzy sets in diagnostic medicine, career determination, and pattern recognition using normalized Hamming distance measureproposed in [23, 24, 27].

BRIEF NOTE ON INTUITIONISTIC FUZZY SETS

Definition 1 [30]:Let X be a nonempty set. A fuzzy set A drawn from X is defined as
$A = \{\langle x, \mu_A(x)\rangle : x \in X\}$,
where $\mu_A(x): X \longrightarrow [0,1]$ is the membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership i.e. having degrees of membership.
Definition 2 [2, 8]:Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x), \nu_A(x): X \longrightarrow [0,1]$define respectively, the

degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Furthermore, we have $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$ i.e.,$\pi_A(x): X \longrightarrow [0,1]$ and $0 \leq \pi_A \leq 1$ for every $x \in X$. $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

Note:

1. Fuzzy set is to intuitionistic fuzzy set what fuzzy multiset is to intuitionistic fuzzy multisets.

2. Every fuzzy set is an intuitionistic fuzzy set, but the reverse is not true.

3. $\mu_A(x) + \nu_A(x) + \pi_A(x) = 1$.

SOME DISTANCE MEASURES IN INTUITIONISTIC FUZZY SETS

Definition 3:Let X be nonempty such that IFS $A, B, C \in X$. Then the distance measure $d$between IFS A and B is a mapping $d: X \times X \to [O, 1]$; if $d(A,B)$ satisfies the following axioms:

$A1 \quad 0 \leq d(A,B) \leq 1$

$A2 \quad d(A,B) = 0$ if and only if $A = B$

$A3 \quad d(A,B) = d(B,A)$

$A4 \quad d(A,C) + d(B,C) \geq d(A,B)$;

$A5$ if $A \subseteq B \subseteq C$, then $d(A,C) \geq d(A,B)$ and $d(A,C) \geq d(B,C)$.

Distance measure is a term that describes the difference between intuitionistic fuzzy sets and can be considered as a dual concept of similarity measure. We make use of the four distance measures proposed in [23, 24, 27] between intuitionistic fuzzy sets, which were partly based on the geometric interpretation of intuitionistic fuzzy sets, and have some good geometric properties.

Let $A = \{\langle x, \mu_A(x_i), \nu_A(x_i), \pi_A(x_i)\rangle : x \epsilon X\}$ and $B = \{\langle x, \mu_B(x_i), \nu_B(x_i), \pi_B(x_i)\rangle : x \epsilon X\}$ be two IFSs in $X = \{x_1, x_2, \ldots, x_n\}$, $i = 1, 2, \ldots, n$. Based on the geometric interpretation of IFS, Szmidt and Kacprzyk [23, 24] proposed the following four distance measures between A and B:

The Hamming distance;

$$d_H(A,B) = \frac{1}{2}\sum_{i=1}^{n}\left(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)| \right) \ldots\ldots 1$$

The Euclidean distance;

$$d_E(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}[(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2]} \ldots\ldots 2$$

The normalized Hamming distance;

$$d_{n-H}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|\nu_A(x_i) - \nu_B(x_i)\right| + \left|\pi_A(x_i) - \pi_B(x_i)\right|)\ldots\ldots 3$$

The normalized Euclidean distance ;

$$d_{n-E}(A,B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}[(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2]}\ldots\ldots 4$$

Example

Let $A = \{\langle 0.6,0.2,0.2\rangle, \langle 0.5,0.3,0.2\rangle\}$ and $B = \{\langle 0.5,0.4,0.1\rangle, \langle 0.4,0.1,0.5\rangle\}$ be IFSs in X such that $X = \{x_1, x_2\}$. We use the above distance measures to calculate the distance between A and B.

Hamming distance of A and B i.e. $d_H(A,B) = 0.25$, Euclidean distance of A and B i.e. $d_E(A,B) = 0.2189$, normalized Hamming distance of A and B i.e. $d_{n-H}(A,B) = 0.125$ and normalized Euclidean distance of A and B i.e. $d_{n-E}(A,B) = 0.1548$.

From these results, we saw that the normalized Hamming distance gives the best distance measure between A and B. This is because the distance is the shortest or smallest. For this reason, we shall make use of normalized Hamming distance in the applications for its high rate of confidence in terms of accuracy.

MODEL OF INTUITIONISTIC FUZZY SETS (IFSS) IN DIAGNOSTIC MEDICINE

Most human reasoning involves the use of variables whose values are uncertain i.e. fuzzy in nature. This is the basis for the concept of linguistic variable, that is, a variable with words values rather than numbers. But in some cases like medical diagnosis, the description by a linguistic variable in terms of membership function alone is not sufficient because there is a chance of the existing of non-membership function. In such a case, intuitionistic fuzzy set (IFS) is suitable because it uses membership function, non-membership function, and the hesitation margin function involved in an uncertain situation.

Let $P = \{p_1, p_2, p_3, p_4\}$ be a set of patients, $D = \{$viral fever, tuberculosis, typhoid, throat disease$\}$ be a set of diseases and $S = \{$temperature, cough, throat pain, headache, body pain$\}$ be a set of symptoms.

Table 1 below is an assumed database of diseases and their symptoms based on medical knowledge in intuitionistic fuzzy nature.

Table 1: Diseases vs Symptoms

	Temperature	Cough	Throat pain	Headache	Body pain
Viral fever	(0.8,0.1,0.1)	(0.2,0.7,0.1)	(0.3,0.5,0.2)	(0.5,0.3,0.2)	(0.5,0.4,0.1)
Tuberculosis	(0.2,0.7,0.1)	(0.9,0.0,0.1)	(0.7,0.2,0.1)	(0.6,0.3,0.1)	(0.7,0.2,0.1)
Typhoid	(0.5,0.3,0.2)	(0.3,0.5,0.2)	(0.2,0.7,0.1)	(0.2,0.6,0.2)	(0.4,0.4,0.2)
Throat disease	(0.1,0.7,0.2)	(0.3,0.6,0.1)	(0.8,0.1,0.1)	(0.1,0.8,01)	(0.1,0.8,0.1)

In Table 1, each symptom S_i is described by three numbers i.e. membership μ, non-membership $\nu$and hesitation margin π.

For the diagnosis sake, we assumed samplesare taken from the patients and analysed. From the analysis, we get Table 2 below.

Table 2: Patients vs Symptoms

	Temperature	Cough	Throat pain	Headache	Body pain
p_1	(0.65,0.15,0.2)	(0.35,0.45,0.2)	(0.15,0.7,0.15)	(0.55,0.35,0.1)	(0.25,0.5,0.25)
p_2	(0.35,0.45,0.2)	(0.65,0.2,0.15)	(0.55,0.3,0.15)	(0.45,0.5,0.05)	(0.75,0.15,0.1)
p_3	(0.15,0.65,0.2)	(0.25,0.3,0.45)	(0.75,0.05,0.2)	(0.25,0.65,0.1)	(0.35,0.55,0.1)
p_4	(0.45,0.4,0.15)	(0.35,0.4,0.25)	(0.15,0.65,0.2)	(0.55,0.35,0.1)	(0.45,0.5,0.05)

Using the normalized Hamming distance aforementioned, to calculate the distance between each of the patients in Table 2 and each of the diseases in Table 1 with respect to each of the symptoms, we get the following tables as shown below.

Table 3: Distance between Patients and Diseases

	Viral fever	Tuberculosis	Typhoid	Throat disease
p_1	3.8×10^{-2}	8.6×10^{-2}	3.0×10^{-2}	8.4×10^{-2}
p_2	6.6×10^{-2}	3.6×10^{-2}	6.0×10^{-2}	7.6×10^{-2}
p_3	8.0×10^{-2}	6.4×10^{-2}	6.0×10^{-2}	3.6×10^{-2}
p_4	4.0×10^{-2}	7.0×10^{-2}	3.2×10^{-2}	8.0×10^{-2}

From Table 3, patient p_1 is diagnosed with typhoid, patient p_2 is diagnosed with tuberculosis, patient p_3 is diagnosed with throat disease and patientp_4 is diagnosed with typhoid.

Note:If the distance between a patient and a particular disease is the shortest, the patient is likely to have the disease.

MODEL OF INTUITIONISTIC FUZZY SETS IN CAREER DETERMINATION

The essence of providing adequate information to students for proper career choice cannot be overemphasized. This is paramount because the numerous problems of lack of proper career guide faced by students are of great consequence on their career choice and efficiency. Therefore, it is expedient that students be given sufficient information on career determination or choice to enhance adequate planning, preparation and proficiency. Among the career determining factors such as academic performance, interest, personality make-up etc; the first mentioned seems to be overriding. We use intuitionistic fuzzy sets as tool since it incorporates the membership degree (i.e. the marks of the questions answered by the student), the non-membership degree (i.e. the marks allocated to the questions the student failed) and the hesitation degree (which is the mark allocated to the questions the student do not attempt).

Let $S = \{s_1, s_2, s_3, s_4\}$ be the set of students, $C =$ {medicine, pharmacy, surgery, anatomy} be the set of careers and $Su =${English Language, Mathematics, Biology, Physics, Chemistry} be the set of subjects related to the careers. We assume the above students sit for examinations (i.e. over 100 marks total) on the above mentioned subjects to determine their career placements and choices. The table below shows careers and related subjects requirements in terms of intuitionistic fuzzy values.

Table 4: Careers vs Subjects

	English Language	Mathematics	Biology	Physics	Chemistry
medicine	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.9,0.0,0.1)	(0.6,0.3,0.1)	(0.8,0.1,0.1)
pharmacy	(0.9,0.1,0.0)	(0.8,0.1,0.1)	(0.8,0.1,0.1)	(0.5,0.3,0.2)	(0.7,0.2,0.1)
surgery	(0.5,0.3,0.2)	(0.5,0.2,0.3)	(0.9,0.0,0.1)	(0.5,0.4,0.1)	(0.7,0.1,0.2)
anatomy	(0.7,0.2,0.1)	(0.5,0.4,0.1)	(0.9,0.1,0.0)	(0.6,0.3,01)	(0.8,0.0,0.2)

Each performance is described by three numbers i.e. membership μ, non-membership $\nu$and hesitation margin π. After the various examinations, the students obtained the following marks as shown in the table below.

Table 5: Students vs Subjects

	English Language	Mathematics	Biology	Physics	Chemistry
s_1	(0.6,0.3,0.1)	(0.5,0.4,0.1)	(0.6,0.2,0.2)	(0.5,0.3,0.2)	(0.5,0.5,0.0)
s_2	(0.5,0.3,0.2)	(0.6,0.2,0.2)	(0.5,0.3,0.2)	(0.4,0.5,0.1)	(0.7,0.2,0.1)
s_3	(0.7,0.1,0.2)	(0.6,0.3,0.1)	(0.7,0.1,0.2)	(0.5,0.4,0.1)	(0.4,0.5,0.1)
s_4	(0.6,0.4,0.0)	(0.8,0.1,0.1)	(0.6,0.0,0.4)	(0.6,0.3,0.1)	(0.5,0.3,0.2)

Using 3 above (since 3 has high rate of accuracy) to calculate the distance between each student and each career with reference to the subjects, we get the table below.

Table 6: Students vs Careers

	medicine	pharmacy	surgery	anatomy
s_1	4.8×10^{-2}	4.4×10^{-2}	4.4×10^{-2}	4.0×10^{-2}
s_2	4.4×10^{-2}	4.4×10^{-2}	2.8×10^{-2}	4.8×10^{-2}
s_3	3.6×10^{-2}	3.6×10^{-2}	4.0×10^{-2}	4.0×10^{-2}
s_4	4.0×10^{-2}	3.6×10^{-2}	4.4×10^{-2}	4.8×10^{-2}

From the above table, the shortest distance gives the proper career determination. s_1 is to read anatomy (anatomist), s_2 is to read surgery (surgeon), s_3 is to read either medicine or pharmacy (doctor or pharmacist), and s_4 is to read pharmacy (pharmacist).

MODEL OF INTUITIONISTIC FUZZY SETS IN PATTERN RECOGNITION

In this process, a sets of patterns is given (intuitionistic in nature), and another unknown pattern called sample is given (also intuitionistic in nature). Both the set of the pattern and that of the sample are within the same feature space or attributes, n. The task is to find the distance between each of the patterns and the sample. The smallest or shortest distance between any of the patterns and the sample shows that, the sample belongs to that pattern. This is what pattern recognition is all about.

Assume that there exist n patterns given by$A_j = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle : x_i \in X\}$,$i = 1,2, \ldots, n$, and$A_j = \{A_1, A_2, \ldots, A_m\}$, for $m \in \mathbb{N}$. Suppose that there is a sample to be recognized, that is $B = \{\langle x_i, \mu_B(x_i), \nu_B(x_i)\rangle : x_i \in X\}$, for $i = 1,2, \ldots, n$.

For example, let six patterns be represented by IFSs in $X = \{x_1, x_2, x_3, x_4\}$ (i.e. feature space for $n = 4$) as

$A_1 = \{\langle 1.0, 0.0, 0.0\rangle, \langle 0.8, 0.1, 0.1\rangle, \langle 0.7, 0.2, 0.1\rangle, \langle 0.5, 0.3, 0.2\rangle\}$,

$A_2 = \{\langle 0.8, 0.1, 0.1\rangle, \langle 1.0, 0.0, 0.0\rangle, \langle 0.9, 0.1, 0.0\rangle, \langle 0.6, 0.1, 0.3\rangle\}$,

$A_3 = \{\langle 0.6, 0.2, 0.2\rangle, \langle 0.8, 0.0, 0.2\rangle, \langle 1.0, 0.0, 0.0\rangle, \langle 0.8, 0.1, 0.1\rangle\}$,

$A_4 = \{\langle 0.8, 0.2, 0.0\rangle, \langle 0.7, 0.2, 0.1\rangle, \langle 0.8, 0.2, 0.0\rangle, \langle 0.5, 0.3, 0.2\rangle\}$,

$A_5 = \{\langle 0.6, 0.3, 0.1\rangle, \langle 0.9, 0.1, 0.0\rangle, \langle 1.0, 0.0, 0.0\rangle, \langle 0.8, 0.0, 0.2\rangle\}$ and

$A_6 = \{\langle 0.9, 0.1, 0.0\rangle, \langle 0.8, 0.1, 0.1\rangle, \langle 0.7, 0.2, 0.1\rangle, \langle 0.5, 0.3, 0.2\rangle\}$

be the classification of building materials. Consider another kind of unknown building material$B$ as $B = \{\langle 0.5, 0.3, 0.2\rangle, \langle 0.6, 0.2, 0.2\rangle, \langle 0.8, 0.1, 0.1\rangle, \langle 0.9, 0.1, 0.0\rangle\} \forall\, A, B \in X$. Our task is to show which class of A_j(for $j = 1,2,3,4,5,6$), the unknown pattern B belongs to. Using 3, we have the following results: $d_{n-H}(A_1, B) = 0.075$, $d_{n-H}(A_2, B) = 0.0688$, $d_{n-H}(A_3, B) = 0.0375$, $d_{n-H}(A_4, B) = 0.0563$, $d_{n-H}(A_5, B) = 0.05$, $d_{n-H}(A_6, B) = 0.0688$. From these results, we see that, the distance between A_3 and B is the smallest, and the distance between A_1 and B is the greatest. Since A_3 approaches B, we say that the unknown pattern B belongs to A_3.

CONCLUSION

Obviously, the idea of intuitionistic fuzzy sets is of immense significance in decision mathematics because it captures all the possibilities involve in real life decision problems.

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