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SUFFICIENT CONDITION FOR A MATRIX TO BE DIAGONALIZABLE

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Abstract In this paper, a sufficient condition for a matrix to be diagonalizable, in the terms of Adjoint is determined and rank of Adjoint of a Matrix $A - \lambda I$ is either 0 or 1 according as λ is repeated or non-repeated Eigen value of Symmetric matrix A. A counter example for a non- diagonalizable matrix is also provided.
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1. INTRODUCTION

One of the famous problems in Linear Algebra is to determine whether a given matrix is diagonalizable or not. From [1], we have "A $n \times n$ matrix A is diagonalizable if and only if it has n Linear Independent vectors". This approach is based on multiplicity of Eigen-Value with their respected Eigen-Vectors. Here, we determined sufficient condition for a matrix to be diagonalizable in terms of rank of Adjoint with special case for multiplicity of Eigen-Value. The result is also study for a symmetric matrix.

2. MAIN RESULTS

Theorem 2.1:- Prove that rank of Adjoint matrix of a diagonal matrix whose all diagonal elements are non- zero is the number of row of given diagonal matrix.

Proof: - Let $A = [a_{ij}]_{n\times n}$ be a diagonal matrix, therefore $|A| = \prod_{i=1}^{n} a_{ii}$

=>$|A| \neq 0$ $because\ a_{ii} \neq 0\ for\ 1 \leq i \leq n$

We know that $|adjA| = |A|^{n-1}$ when A is non-singular.

Therefore $|adjA| \neq 0$ =>$\rho(adjA) = n$.

Theorem 2.2:- Prove that rank of Adjoint matrix of a diagonal matrix whose all diagonal elements are non- zero except exactly one is one.

Proof: - Let A be diagonal matrixes with all diagonal elements are non- zero except exactly one, then A is defined as $A = [a_{ij}]_{n\times n}$ where $a_{ij} = \begin{cases} 0 & if\ i \neq j \\ a_{ii} & if\ i = j \neq k \\ 0 & if\ i = j = k \end{cases}$

Now, we defines$B = [A_{ij}]_{n\times n}$, where $A_{ij} = \begin{cases} 0 & if\ i \neq j \\ 0 & if\ i = j \neq k \\ \prod_{\substack{i=1 \\ i\neq k}}^{n} a_{ii} & if\ i = j = k \end{cases}$ are cofactor of a_{ij}.

Here B has only one non-zero entry, hence $\rho(B) = 1$

We know that $adj(A) = B^T$ => $\rho\big(adj(A)\big) = \rho(B)$ => $\rho\big(adj(A)\big) = 1$

Corollary: - Prove that Adjoint matrix of a diagonal matrix whose two or more diagonal entries are zero is Null Matrix.

Proof: - If we assume that one more diagonal entry is zero, say a_{ss} where $s \neq k$, then B becomes a Null Matrix.

Theorem 2.3: - A is a diagonalizable matrix if and only if $A - \lambda I$ diagonalizable.

Proof: - Condition is Necessary:-

If A is diagonalizable matrix, then there exist a non-singular matrix P such that $D = P^{-1}AP$ where D is a diagonal matrix with diagonal elements are Eigen values of A.

Also $I = P^{-1}IP$

Then $P^{-1}(A - \lambda I)P = P^{-1}AP$ -$\lambda P^{-1}IP = D - \lambda I$, hence we get a diagonal matrix with diagonal elements are Eigen values of A are diminished by λ.

Hence we get $A - \lambda I$ diagonalizable

Condition is Sufficient:-

If $A - \lambda I$ is diagonalizable matrix, then there exist a non-singular matrix Q such that $D = Q^{-1}(A - \lambda I)Q$ where D is a diagonal matrix with diagonal elements are Eigen values of $A - \lambda I$.

Also $I = Q^{-1}IQ$

Then $Q^{-1}AQ = Q^{-1}(A - \lambda I + \lambda I)Q = Q^{-1}(A - \lambda I)Q + \lambda Q^{-1}IQ = D + \lambda I$, hence we get a diagonal matrix with diagonal elements are Eigen values of A.

Hence we get A diagonalizable.

Theorem 2.4:- A matrix A can be diagonalizable only if Adjoint Matrix of a Matrix $A - \lambda I$ is a zero matrix for every repeated Eigen value λ.

Proof:- If A is a diagonalizable matrix with λ is repeated Eigen value. By use of Theorem 2.3 $A - \lambda I$ must be diagonalizable. Also repeated Eigen value λ of A gives zero as repeated Eigen value of $A - \lambda I$. By using corollary Theorem 2.2, Adjoint Matrix of $A - \lambda I$ is a zero matrix.

Now we provide example of a non-diagonalizable matrix which has repeated Eigen Value λ with its Adjoint Matrix of the matrix $A - \lambda I$ is not a Null Matrix, example for this is given below. Here $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ has 0, 0 and 1 where 0 is repeated Eigen Value, then $adjoint\ (A - 0I) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Theorem 2.5:- If matrix A is diagonalizable with λ is non-repeated Eigen Value of A, then rank of Adjoint Matrix of a Matrix $(A - \lambda I)$ is a one.

Theorem 2.6: - Prove that a rank of Adjoint matrix of a Matrix $A - \lambda I$ is either 0 or 1 according as λ is repeated or non-repeated Eigen value of Symmetric matrix A.

Proof: - If A is symmetric matrix, then A is always diagonalizable.

Case 1:- λ is repeated Eigen value of A

If λ is repeated Eigen value of A, then by use of theorem 2.4, then we have Adjoint Matrix of a Matrix $A - \lambda I$ is a zero matrix. Therefore $\rho(A - \lambda I) = 0$

Case 2:- λ is non-repeated Eigen value of A

If λ is non- repeated Eigen value of A, then by use of theorem 2.5, then we have rank of Adjoint Matrix of a Matrix $A - \lambda I$ is one. Therefore $\rho(A - \lambda I) = 1$

3. REFERENCES

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