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An inventory system with a standby server

M. Bhuvaneshwari[1], S. Padmasekaran[2] and K. Jeganathan[3]
[1]PG Assistant in Mathematics,
Government Higher Secondary School, Sathirakudi,
Ramanathapuram, India
[2]Department of Mathematics, Periyar University, India
[3]Ramanujan Institute for Advanced Study in Mathematics,
University of Madras, Chennai, India.

Abstract

In this paper, we work on (s, Q) inventory queuing system for providing two types of customers services, namely high priority customer and low priority customer, with one basic server interruption and a standby server replaces whenever the basic server gets interruption. The high priority customer demands a unit item along with some services and the low priority customer demands only the services (on the used item) but not purchasing the item at that time. The high priority customers are allowed in the queue with finite capacity waiting hall and low priority customers allowed in the finite orbit with retrial policy when the server is busy, otherwise, the customer is lost. Also, in this discussion, the server can go in orbit search of customers immediately after either the inventory is stocked out or there is no queue. The customers' arrivals of both types follow Poisson processes and the times of both the services follow the exponential distributions while the interruption times and the stand-by service times are exponentially distributed. Finally, the system performance measures are derived in the steady states.

Keywords: Continuous review, Perishable commodity, Heterogeneous servers, Standby server.

1 Introduction

In many real life situations, the demanded item cannot be delivered immediately due to the installation or demonstration or the upgrading some facilities and hence we require some positive service time, eventually this leads to build a waiting hall for the customers. Berman and Kim [1] analyzed an inventory queuing problem under the assumption of arrival of customers follows a Poisson process and the service times are exponentially distributed in which mean inter arrival time is larger than the mean service time. Berman and Sapna [2] studied an inventory queuing problem under the assumptions of Poisson arrivals, arbitrarily distributed service times, zero lead times and finite capacity of waiting room. They determine the optimal ordering quantity based on the given cost structure, derived from the minimum long run expected cost per unit time. Further, Elango [3], he studied the Markovian inventory system along with service facility and instantaneous replenishment of orders. The service time assumed to be exponential distribution with the rate depending on the queue length. Arivarignan et al. [4] studied the same problem with exponential lead time. In Sivakumar and Arivarignan [5], they worked the inventory problem with exponential service and lead times but the demand of an item is arbitrarily distributed.

Also, when the server is busy, the customers may allow in a retrial orbit and repeated attempt to capture the free server after a random amount of time. This model is known as a retrial queuing model. This is extensively studied by many authors. With the reference of the books of Falin and Templeton [6] and Artalejo and Gomez Corral [7], they analyze the both theory and applications on retrial queues which is useful to the readers. In Artalejo et. al [8], first studied this inventory model with positive lead time for retrying the orbiting customer to get unsatisfied demand in algorithmic approach. Ushakumari [9] derived the analytical solution to the same model. In many situations we can find out the different classes of people to get the different quality of services. Under non-preemptive discipline, high priority customer is allowed in a queue when a low priority customer is in the server and low priority customer goes into a retrial orbit when the server is busy. A non pre-emptive retrial queuing model is first investigated by Choi and Park [10]. They allowed the retrial queue for both priority and ordinary customers under the priority customer have non pre-emptive priority over ordinary customers and queued in FCFS. In Krishnamoorthy and Jose [11], the authors studied the inventory (s, S) system with an orbit size infinite. They considered a waiting hall for fresh customers with finite capacity and the orbit used only for retrial customers. The inflow and outflow rate of retrying customers in the orbit and the length of queue are considered to be independent. In Krishnamoorthy and Islam [12], Sivakumar and Arivarignan [13] and Paul Manuel et al. [14], all these inventory retrail queuing model, the selection time between the pooled customers is according to exponential distribution.

In this paper, we work on a continuous (s, S) inventory - queuing system with high priority and low priority customers. The arrivals and services of both the customers are heterogeneous types. One basic server and one standby server are used in the model. A finite capacity waiting hall and a finite capacity orbit queue are respectively used for high and low priorty customers. Orbit search used for low priority customers with non-pre-emptive priority service policy. This helps to minimize the ideal time of the server.

The paper is organized as follows. In section 2, the mathematical model and the notations are defined. Section 3 gives the analysis of the model and the steady state solution of the model. The various system performance measures are given in section 4.

2 Mathematical Model and Notations

In this model, we consider the high priority customers(HPC) who purchase the product along with services which may include the installation or demonstration of how to use the product or upgrading the additional facilities and the low priority customers(LPC) is only provided the services which includes the maintenance, or repair or upgrading but not purchasing the product. We observe that the number of arrivals of both high and low priority customers follow Poisson processes and servicing times of both high and low priority customers are exponentially distributed. Let λ_1 and μ_1 are the arrival rate and service rate of the high priority customers and λ_2 and μ_2 are the arrival rate and service rate of the low priority customers respectively.

In this model, we discuss the two types of servers, namely basic server and standby server. We assume that the interruption may occur only the basic server with the rate η and the completion of interruption with the rate α. The times of both these occurrences follow exponential distributions. We assume that the basic server is only providing the services till the interruption occurs. The standby server can also be used when the basic server has an interruption.

In this model, the waiting hall with finite capacity, say N, is only used for high priority customers. The high priority customer is lost when the hall is full.

In this model, retrial policy only used for low priority customers in the finite orbit, say M. Assume the inter-retrial times of repeated attempts of the low priority customer demand for free server follows the exponential distribution with the rate θ, the corresponding rate of demand of customers in the orbit is jθ when the orbit size is j. This is known as the classical retrial policy.

In this model, under non-pre-emptive priority service policy, the basic server can also search the orbiting customer for providing service with the probability r and 1-r is the corresponding probability of ideal of basic server. This is the case whenever there is no customer in the waiting hall or the inventories are stocked out.

In this model, we work on a continuous review inventory system and the ordering policy is (s, S). The small letter s denotes the reorder level and the capital letter S denotes the maximum inventory level of the system. Assume q be the fixed order quantity with instantaneous replenishment given by $q = S - s$ and the lead time occur with the rate β, where the lead time is the time gap between the placing the order and receiving the order.

2.1 Notations:

$$\begin{aligned}
I_k &= \text{an identity matrix of order k} \\
\mathbf{e} &= (1, 1, \ldots, 1)^T \\
[C]_{ij} &= \text{entry at } (i,j)^{th} \text{ position of a matrix C} \\
a \in V_i^j &= a = i, i+1, \ldots j \\
\underset{i=r}{\overset{k}{\Omega}} y_i &= \begin{cases} y_r y_{r-1} \cdots y_k & \text{if } r \geq k \\ 1 & \text{if } r < k \end{cases} \\
\delta_{xy} &= \begin{cases} 1 & \text{if } \quad x = y \\ 0 & \text{otherwise} \end{cases} \\
\bar{\delta_{xy}} &= 1 - \delta_{xy} \\
H(a) &= \begin{cases} 1 & \text{if } \ a \geq 0 \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$

3 Analysis of the Model

At time $t \geq 0$, the state of the system can be expressed by the stochastic process $y(t) = \{(y_1(t), y_2(t), y_3(t), y_4(t))\}$, where $y_1(t)$ means inventory level at time t, $y_2(t)$ means server's status at time t, it is defined as follows:

$$y_2(t) = \begin{cases} 0, & \text{if Both servers are free at time t,} \\ 1, & \text{if Basic server is busy with HPC and standby server is free at time t,} \\ 2, & \text{if Basic server is on interruption and standby server is busy with HPC at time t,} \\ 3, & \text{if Basic server is busy with LPC and standby server is free at time t,} \\ 4, & \text{if Basic server is on interruption and standby server is busy with LPC at time t,} \\ 5, & \text{if Basic server is on interruption and standby server is free at time t,} \end{cases}$$

$y_3(t)$ means number of HPC in the queue at time t and $y_4(t)$ means number of LPC in the queue at time t. The state space of the model is $A = a_1 \cup a_2 \cup a_3 \cup a_4 \cup a_5 \cup a_6 \cup a_7 \cup a_8 \cup a_9 \cup a_{10}$, where

$$\begin{aligned}
a_1 &= \{(0, 0, j_3, j_4) \mid 0 \leq j_3 \leq N; 0 \leq j_4 \leq M\}, \\
a_2 &= \{(j_1, 0, 0, j_4) \mid 1 \leq j_1 \leq S; 0 \leq j_4 \leq M\}, \\
a_3 &= \{(j_1, 1, j_3, j_4) \mid 1 \leq j_1 \leq S; 0 \leq j_3 \leq N; 0 \leq j_4 \leq M\}, \\
a_4 &= \{(j_1, 2, j_3, j_4) \mid 1 \leq j_1 \leq S; 0 \leq j_3 \leq N; 0 \leq j_4 \leq M\}, \\
a_5 &= \{(0, 3, j_3, j_4) \mid 0 \leq j_3 \leq N; 0 \leq j_4 \leq M\}, \\
a_6 &= \{(j_1, 3, j_3, j_4) \mid 1 \leq j_1 \leq S; 0 \leq j_3 \leq N; 0 \leq j_4 \leq M\}, \\
a_7 &= \{(0, 4, j_3, j_4) \mid 0 \leq j_3 \leq N; 0 \leq j_4 \leq M\}, \\
a_8 &= \{(j_1, 4, j_3, j_4) \mid 1 \leq j_1 \leq S; 0 \leq j_3 \leq N; 0 \leq j_4 \leq M\}, \\
a_9 &= \{(0, 5, j_3, j_4) \mid 0 \leq j_3 \leq N; 0 \leq j_4 \leq M\}, \\
a_{10} &= \{(j_1, 5, 0, j_4) \mid 1 \leq j_1 \leq S; 0 \leq j_4 \leq M\}
\end{aligned}$$

The set of states in the level $(\mathbf{0})$ is denoted by $(\mathbf{0}) = a_1 \cup a_5 \cup a_7 \cup a_9$. The set of states in the level $(\mathbf{j_1})$, $1 \leq j_1 \leq S$, is denoted by $(\mathbf{j_1}) = a_2 \cup a_3 \cup a_4 \cup a_6 \cup a_8 \cup a_{10}$.

Then the rate matrix of $y(t) = \{(y_1(t), y_2(t), y_3(t), y_4(t))\}$ is given by

$$\Gamma = \begin{array}{c} (S) \\ (S-1) \\ \vdots \\ (s+1) \\ (s) \\ (s-1) \\ \vdots \\ (2) \\ (1) \\ (0) \end{array} \begin{pmatrix} \Omega_2 & \Delta_2 & & & & & & & & & \\ & \Omega_2 & \Delta_2 & & & & & & & & \\ & & \cdots & & & & & & & & \\ & & & \cdots & \Omega_2 & \Delta_2 & & & & & \\ \Psi & & & & & \Omega_1 & \Delta_2 & & & & \\ & \Psi & & & & & \Omega_1 & & & & \\ & & \cdots & & & & \cdots & & & & \\ & & \Psi & & & & & \cdots & \Omega_1 & \Delta_2 & \\ & & & \Psi & & & & & & \Omega_1 & \Delta_1 \\ & & & & \Psi_1 & & & & & & \Omega_0 \end{pmatrix}$$

where the matrices Δ_1, Ψ_1 and Ω_0 are of dimention $(M+1)(4(N+1)+2) \times 4(N+1)(M+1)$, $4(N+1)(M+1) \times (M+1)(4(N+1)+2)$ and $4(N+1)(M+1) \times 4(N+1)(M+1)$ and other matrices Ω_1, Ω_2, Δ_2 and Ψ are of dimention $(M+1)(4(N+1)+2) \times (M+1)(4(N+1)+2)$.

The elements of the sub-matrices of Γ can be described as followes:

$$\Psi_1 = \begin{cases} \beta, & k_1 = Q, \quad k_2 = j_2, \quad k_3 = j_3, \quad k_4 = j_4, \\ & j_1 = 0, \quad j_2 = 0, \quad j_3 = 0, \quad j_4 \in V_1^M, \\ & \\ & k_1 = Q, \quad k_2 = 1, \quad k_3 = j_3 - 1, \quad k_4 = j_4, \\ & j_1 = 0, \quad j_2 = 0, \quad j_3 \in V_1^N, \quad j_4 \in V_0^M, \\ & \\ & k_1 = Q, \quad k_2 = j_2, \quad k_3 = j_3, \quad k_4 = j_4, \\ & j_1 = 0, \quad j_2 \in V_3^5, \quad j_3 \in V_0^N, \quad j_4 \in V_0^M, \\ 0, & \text{otherwise.} \end{cases}$$

$$\Psi = \beta \times I_{(M+1)(4(N+1)+2)}$$

$$\Delta_1 = \begin{cases} \mu_1, & k_1 = 0, \quad k_2 = 0, \quad k_3 = j_3, \quad k_4 = j_4, \\ & j_1 = 1, \quad j_2 = 1, \quad j_3 \in V_0^N, \quad j_4 = 0, \\ & \\ & k_1 = 0, \quad k_2 = 5, \quad k_3 = j_3, \quad k_4 = j_4, \\ & j_1 = 1, \quad j_2 = 2, \quad j_3 \in V_0^N, \quad j_4 \in V_0^M, \\ & \\ r\mu_1, & k_1 = 0, \quad k_2 = 3, \quad k_3 = j_3, \quad k_4 = j_4 - 1, \\ & j_1 = 1, \quad j_2 = 1, \quad j_3 \in V_0^N, \quad j_4 \in V_1^M, \\ & \\ (1-r)\mu_1, & k_1 = 0, \quad k_2 = 0, \quad k_3 = j_3, \quad k_4 = j_4, \\ & j_1 = 1, \quad j_2 = 1, \quad j_3 \in V_0^N, \quad j_4 \in V_1^M, \\ 0, & \text{otherwise.} \end{cases}$$

$$\Delta_2 = \begin{cases}
\mu_1, & k_1=j_1-1, \quad k_2=0, \quad k_3=j_3, \quad k_4=j_4, \\
 & j_1 \in V_2^S, \quad j_2=1, \quad j_3=0, \quad j_4=0, \\[2ex]
 & k_1=j_1-1, \quad k_2=j_2, \quad k_3=j_3-1, \quad k_4=j_4, \\
 & j_1 \in V_2^S, \quad j_2=1, \quad j_3 \in V_1^N, \quad j_4 \in V_0^M, \\[2ex]
 & k_1=j_1-1, \quad k_2=5, \quad k_3=j_3, \quad k_4=j_4, \\
 & j_1 \in V_2^S, \quad j_2=2, \quad j_3=0, \quad j_4 \in V_0^M, \\[2ex]
 & k_1=j_1-1, \quad k_2=j_2, \quad k_3=j_3-1, \quad k_4=j_4, \\
 & j_1 \in V_2^S, \quad j_2=2, \quad j_3 \in V_1^N, \quad j_4 \in V_0^M, \\[2ex]
r\mu_1, & k_1=j_1-1, \quad k_2=3, \quad k_3=j_3, \quad k_4=j_4-1, \\
 & j_1 \in V_2^S, \quad j_2=1, \quad j_3=0, \quad j_4 \in V_1^M, \\[2ex]
(1-r)\mu_1, & k_1=j_1-1, \quad k_2=0, \quad k_3=j_3, \quad k_4=j_4, \\
 & j_1 \in V_2^S, \quad j_2=1, \quad j_3=0, \quad j_4 \in V_1^M, \\
0, & \text{otherwise.}
\end{cases}$$

$$\Omega_0 = \begin{cases}
\lambda_1, & k_1=j_1, \quad k_2=j_2, \quad k_3=j_3+1, \quad k_4=j_4, \\
 & j_1=0, \quad j_2=0,3,4,5, \quad j_3 \in V_0^{N-1}, \quad j_4 \in V_0^M, \\[2ex]
\lambda_2, & k_1=j_1, \quad k_2=3, \quad k_3=j_3, \quad k_4=j_4, \\
 & j_1=0, \quad j_2=0, \quad j_3 \in V_0^N, \quad j_4 \in V_0^M, \\[2ex]
 & k_1=j_1, \quad k_2=j_2, \quad k_3=j_3, \quad k_4=j_4+1, \\
 & j_1=0, \quad j_2=3,4, \quad j_3 \in V_0^N, \quad j_4 \in V_0^{M-1}, \\[2ex]
 & k_1=j_1, \quad k_2=4, \quad k_3=j_3, \quad k_4=j_4, \\
 & j_1=0, \quad j_2=5, \quad j_3 \in V_0^N, \quad j_4 \in V_0^M, \\[2ex]
j_4\theta, & k_1=j_1, \quad k_2=3, \quad k_3=j_3, \quad k_4=j_4-1, \\
 & j_1=0, \quad j_2=0, \quad j_3 \in V_0^N, \quad j_4 \in V_1^M, \\[2ex]
 & k_1=j_1, \quad k_2=4, \quad k_3=j_3, \quad k_4=j_4-1, \\
 & j_1=0, \quad j_2=5, \quad j_3 \in V_0^N, \quad j_4 \in V_1^M, \\[2ex]
\mu_2, & k_1=j_1, \quad k_2=0, \quad k_3=j_3, \quad k_4=j_4, \\
 & j_1=0, \quad j_2=3, \quad j_3 \in V_0^N, \quad j_4=0, \\[2ex]
 & k_1=j_1, \quad k_2=5, \quad k_3=j_3, \quad k_4=j_4, \\
 & j_1=0, \quad j_2=4, \quad j_3 \in V_0^N, \quad j_4 \in V_0^M, \\[2ex]
r\mu_2, & k_1=j_1, \quad k_2=j_2, \quad k_3=j_3, \quad k_4=j_4-1, \\
 & j_1=0, \quad j_2=3, \quad j_3 \in V_0^N, \quad j_4 \in V_1^M, \\[2ex]
(1-r)\mu_2, & k_1=j_1, \quad k_2=0, \quad k_3=j_3, \quad k_4=j_4, \\
 & j_1=0, \quad j_2=3, \quad j_3 \in V_0^N, \quad j_4 \in V_1^M,
\end{cases}$$

$$
\begin{cases}
\eta, & \begin{array}{llll} k_1 = j_1, & k_2 = 4, & k_3 = j_3, & k_4 = j_4, \\ j_1 = 0, & j_2 = 3, & j_3 \in V_0^N, & j_4 \in V_0^M, \end{array} \\
\alpha, & \begin{array}{llll} k_1 = j_1, & k_2 = 3, & k_3 = j_3, & k_4 = j_4, \\ j_1 = 0, & j_2 = 4, & j_3 \in V_0^N, & j_4 \in V_0^M, \end{array} \\
 & \begin{array}{llll} k_1 = j_1, & k_2 = 0, & k_3 = j_3, & k_4 = j_4, \\ j_1 = 0, & j_2 = 5, & j_3 \in V_0^N, & j_4 = 0, \end{array} \\
r\alpha, & \begin{array}{llll} k_1 = j_1, & k_2 = 3, & k_3 = j_3, & k_4 = j_4 - 1, \\ j_1 = 0, & j_2 = 5, & j_3 \in V_0^N, & j_4 \in V_1^M, \end{array} \\
(1-r)\alpha, & \begin{array}{llll} k_1 = j_1, & k_2 = 0, & k_3 = j_3, & k_4 = j_4, \\ j_1 = 0, & j_2 = 5, & j_3 \in V_0^N, & j_4 \in V_1^M, \end{array} \\
\begin{array}{l} -(\bar{\delta}_{j_3 N}\lambda_1 + \lambda_2 + \beta \\ +j_4\theta + \alpha\delta_{j_2 5}), \end{array} & \begin{array}{llll} k_1 = j_1, & k_2 = j_2, & k_3 = j_3, & k_4 = j_4, \\ j_1 = 0, & j_2 = 0, 5, & j_3 \in V_0^N, & j_4 \in V_0^M, \end{array} \\
\begin{array}{l} -(\bar{\delta}_{j_3 N}\lambda_1 + \bar{\delta}_{j_3 M}\lambda_2 + \beta \\ +\mu_2 + \eta\delta_{j_2 3} + \alpha\delta_{j_2 4}), \end{array} & \begin{array}{llll} k_1 = j_1, & k_2 = j_2, & k_3 = j_3, & k_4 = j_4, \\ j_1 = 0, & j_2 = 3, 4, & j_3 \in V_0^N, & j_4 \in V_0^M, \end{array} \\
0, & \text{otherwise.}
\end{cases}
$$

For $k = 1, 2$

$$
\Omega_k = \begin{cases}
\lambda_1, & \begin{array}{llll} k_1 = j_1, & k_2 = 1, & k_3 = j_3, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 0, & j_3 = 0, & j_4 \in V_0^M, \end{array} \\
 & \begin{array}{llll} k_1 = j_1, & k_2 = j_2, & k_3 = j_3 + 1, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 \in V_1^4, & j_3 \in V_0^{N-1}, & j_4 \in V_0^M, \end{array} \\
 & \begin{array}{llll} k_1 = j_1, & k_2 = 2, & k_3 = j_3, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 5, & j_3 = 0, & j_4 \in V_0^M, \end{array} \\
\lambda_2, & \begin{array}{llll} k_1 = j_1, & k_2 = 3, & k_3 = j_3, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 0, & j_3 = 0, & j_4 \in V_0^M, \end{array} \\
 & \begin{array}{llll} k_1 = j_1, & k_2 = j_2, & k_3 = j_3, & k_4 = j_4 + 1, \\ j_1 \in V_1^S, & j_2 \in V_1^4, & j_3 \in V_0^N, & j_4 \in V_0^{M-1}, \end{array} \\
 & \begin{array}{llll} k_1 = j_1, & k_2 = j_2, & k_3 = j_3, & k_4 = j_4 + 1, \\ j_1 \in V_1^S, & j_2 = 5, & j_3 = 0, & j_4 \in V_0^{M-1}, \end{array}
\end{cases}
$$

$$
\begin{cases}
j_4\theta, & \begin{array}{llll} k_1 = j_1, & k_2 = 3, & k_3 = j_3, & k_4 = j_4 - 1, \\ j_1 \in V_1^S, & j_2 = 0, & j_3 = 0, & j_4 \in V_0^M, \end{array} \\
 & \begin{array}{llll} k_1 = j_1, & k_2 = 4, & k_3 = j_3, & k_4 = j_4 - 1, \\ j_1 \in V_1^S, & j_2 = 5, & j_3 = 0, & j_4 \in V_1^M, \end{array} \\
\eta & \begin{array}{llll} k_1 = j_1, & k_2 = 2, & k_3 = j_3, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 1, & j_3 \in V_0^N, & j_4 \in V_0^M, \end{array} \\
 & \begin{array}{llll} k_1 = j_1, & k_2 = 4, & k_3 = j_3, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 3, & j_3 \in V_0^N, & j_4 \in V_0^M, \end{array} \\
\mu_2, & \begin{array}{llll} k_1 = j_1, & k_2 = 0, & k_3 = j_3, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 3, & j_3 = 0, & j_4 = 0, \end{array} \\
 & \begin{array}{llll} k_1 = j_1, & k_2 = 1, & k_3 = j_3 - 1, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 3, & j_3 \in V_1^N, & j_4 \in V_0^M, \end{array} \\
 & \begin{array}{llll} k_1 = j_1, & k_2 = 5, & k_3 = j_3, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 4, & j_3 = 0, & j_4 \in V_0^M, \end{array} \\
 & \begin{array}{llll} k_1 = j_1, & k_2 = 2, & k_3 = j_3 - 1, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 4, & j_3 \in V_1^N, & j_4 \in V_0^M, \end{array} \\
(1-r)\mu_2, & \begin{array}{llll} k_1 = j_1, & k_2 = 0, & k_3 = j_3, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 3, & j_3 = 0, & j_4 \in V_1^M, \end{array} \\
r\mu_2, & \begin{array}{llll} k_1 = j_1, & k_2 = j_2, & k_3 = j_3, & k_4 = j_4 - 1, \\ j_1 \in V_1^S, & j_2 = 3, & j_3 = 0, & j_4 \in V_1^M, \end{array} \\
\alpha, & \begin{array}{llll} k_1 = j_1, & k_2 = 1, & k_3 = j_3, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 2, & j_3 \in V_0^N, & j_4 \in V_0^M, \end{array} \\
 & \begin{array}{llll} k_1 = j_1, & k_2 = 3, & k_3 = j_3, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 4, & j_3 \in V_0^N, & j_4 \in V_0^M, \end{array} \\
 & \begin{array}{llll} k_1 = j_1, & k_2 = 0, & k_3 = j_3, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 5, & j_3 = 0, & j_4 = 0, \end{array} \\
(1-r)\alpha, & \begin{array}{llll} k_1 = j_1, & k_2 = 0, & k_3 = j_3, & k_4 = j_4, \\ j_1 \in V_1^S, & j_2 = 5, & j_3 = 0, & j_4 \in V_1^M, \end{array} \\
r\alpha, & \begin{array}{llll} k_1 = j_1, & k_2 = 3, & k_3 = j_3, & k_4 = j_4 - 1, \\ j_1 \in V_1^S, & j_2 = 5, & j_3 = 0, & j_4 \in V_1^M, \end{array}
\end{cases}
$$

$$\begin{cases} -(\lambda_1+\lambda_2+\delta_{k1}\beta & k_1=j_1, \quad k_2=j_2, \quad k_3=j_3, \quad k_4=j_4,\\ +j_4\theta+\alpha\delta_{j_25}), & j_1\in V_1^S, \quad j_2=0,5, \quad j_3=0, \quad j_4\in V_0^M,\\ \\ -(\bar{\delta}_{j_3N}\lambda_1+\bar{\delta}_{j_4M}\lambda_2+\delta_{k1}\beta & k_1=j_1, \quad k_2=j_2, \quad k_3=j_3, \quad k_4=j_4,\\ +\mu_1+\alpha\delta_{j_21}+\eta\delta_{j_22}), & j_1\in V_1^S, \quad j_2=1,2, \quad j_3\in V_0^N, \quad j_4\in V_0^M,\\ \\ -(\bar{\delta}_{j_3N}\lambda_1+\bar{\delta}_{j_4M}\lambda_2+\delta_{k1}\beta & k_1=j_1, \quad k_2=j_2, \quad k_3=j_3, \quad k_4=j_4,\\ +\mu_2+\alpha\delta_{j_24}+\eta\delta_{j_23}), & j_1\in V_1^S, \quad j_2=3,4, \quad j_3\in V_0^N, \quad j_4\in V_0^M,\\ \\ 0, & \text{otherwise.} \end{cases}$$

3.1 Steady-state Analysis

We can observed from the formation of Γ that the time homogeneous Markov process $y(t)$ on the state space A is irreducible, aperiodic and persistent non-null. Hence the limiting distribution

$$x^{(j_1,j_2,j_3,j_4)} = \lim_{t\to\infty} Pr[y_1(t)=j_1, y_2(t)=j_2, y_3(t)=j_3, y_4(t)=j_4|y_1(0),y_2(0),y_3(0),y_4(0)]$$

exists and is independent of the initial state, that is,

$$\mathbf{x^{(j_1,j_2,j_3)}} = \begin{cases} (x^{(j_1,j_2,j_3,0)}\cdots,x^{(j_1,j_2,j_3,M)}), & j_1=0;\ j_2=0,3,4,5;\ j_3\in V_0^N;\\ (x^{(j_1,j_2,j_3,0)}\cdots,x^{(j_1,j_2,j_3,M)}), & j_1\in V_1^S;\ j_2=0,5;\ j_3=0;\\ (x^{(j_1,j_2,j_3,0)}\cdots,x^{(j_1,j_2,j_3,M)}), & j_1\in V_1^S;\ j_2\in V_1^4;\ j_3\in V_0^N; \end{cases}$$

$$\mathbf{x^{(j_1,j_2)}} = \begin{cases} (x^{(j_1,j_2,0)}), & j_1\in V_1^S;\ j_2=0,5;\\ (x^{(j_1,j_2,0)}\cdots,x^{(j_1,j_2,N)}); & j_1=0;\ j_2=0,3,4,5;\\ (x^{(j_1,j_2,0)}\cdots,x^{(j_1,j_2,N)}); & j_1\in V_1^S;\ j_2\in V_1^4; \end{cases}$$

$$\mathbf{x^{(j_1)}} = \begin{cases} (x^{(j_1,0)},x^{(j_1,3)},x^{(j_1,4)},x^{(j_1,5)}), & j_1=0;\\ (x^{(j_1,0)}\cdots,x^{(j_1,5)}); & j_1\in V_1^S; \end{cases}$$

$$\mathbf{X} = (\mathbf{x^{(0)}},\mathbf{x^{(1)}},\ldots,\mathbf{x^{(S)}})$$

satisfies

$$\boldsymbol{X}\Gamma=\mathbf{0} \quad \text{and} \tag{1}$$

$$\sum\sum\sum\sum_{(j_1,j_2,j_3,j_4)}\phi^{(j_1,j_2,j_3,j_4)}=1 \tag{2}$$

From (1), we can get the following set of equations:

$$\begin{aligned} \mathbf{x^{(j_1)}}\Omega_0+\mathbf{x^{(j_1+1)}}\Delta_1 &= \mathbf{0}, \quad j_1=0,\\ \mathbf{x^{(j_1)}}\Omega_1+\mathbf{x^{(j_1+1)}}\Delta_2 &= \mathbf{0}, \quad j_1=1,2,\ldots,s,\\ \mathbf{x^{(j_1)}}\Omega_2+\mathbf{x^{(j_1+1)}}\Delta_2 &= \mathbf{0}, \quad j_1=s+1,..,Q-1,\\ \mathbf{x^{(0)}}\Psi_1+\mathbf{x^{(j_1)}}\Omega_2+\mathbf{x^{(j_1+1)}}\Delta_2 &= \mathbf{0}, \quad j_1=Q,\\ \mathbf{x^{(j_1-Q)}}\Psi+\mathbf{x^{(j_1)}}\Omega_2+\mathbf{x^{(j_1+1)}}\Delta_2 &= \mathbf{0}, \quad j_1=Q+1,..,S-1,\\ \mathbf{x^{(j_1-Q)}}\Psi+\mathbf{x^{(j_1)}}\Omega_2 &= \mathbf{0}, \quad j_1=S, \end{aligned} \tag{$*$}$$

After extensive computational work, the above equations, (except $(*)$), yield

$$\mathbf{x}^{(\mathbf{j_1})} = \mathbf{x}^{(\mathbf{Q})}\Phi_{j_1}, \; j_1 = 0, 1, \ldots, S.$$

where

$$\Phi_{j_1} = \begin{cases} (-1)^{Q-j_1}(\Delta_2\Omega_2^{-1})^{(Q-(s+1))}(\Delta_2\Omega_1^{-1})^{s}(\Delta_1\Omega_0^{-1}), & j_1 = 0, \\ (-1)^{Q-j_1}(\Delta_2\Omega_2^{-1})^{(Q-(s+1))}(\Delta_2\Omega_1^{-1})^{((s+1)-j_1)}, & j_1 = 1, 2, \ldots, s, \\ (-1)^{Q-j_1}(\Delta_2\Omega_2^{-1})^{(Q-j_1)}, & j_1 = s+1, s+2, \ldots, Q-1, \\ I, & j_1 = Q, \\ \sum\limits_{i=0}^{S-j_1}(-1)^{(2Q+1-j_1)}(\Delta_2\Omega_2^{-1})^{(S+s-(j_1+i+1))}(\Delta_2\Omega_1^{-1})^{(j+1)}(\Psi\Omega_2^{-1}), & \\ & j_1 = Q+1, Q+2, \ldots, S, \end{cases}$$

$\mathbf{x}^{(\mathbf{Q})}$ can be obtained by solving equation $(*)$ and $\mathbf{Xe} = 1$.

That is,

$$\mathbf{x}^{(\mathbf{Q})}\Bigg((-1)^{Q}(\Delta_2\Omega_2^{-1})^{(Q-(s+1))}(\Delta_2\Omega_1^{-1})^{s}(\Delta_1\Omega_0^{-1})\Psi_1 + \Omega_2 + \sum_{j=0}^{s-1}(-1)^{Q}(\Delta_2\Omega_2^{-1})^{(2(s-1)-j)}(\Delta_2\Omega_1^{-1})^{(j+1)}(\Psi\Omega_2^{-1})\Delta_2\Bigg) = \mathbf{0},$$

and

$$\mathbf{x}^{(\mathbf{Q})}\Bigg[(-1)^{Q}(\Delta_2\Omega_2^{-1})^{(Q-(s+1))}(\Delta_2\Omega_1^{-1})^{s}(\Delta_1\Omega_0^{-1}) + \sum_{i=1}^{s}(-1)^{Q-i}(\Delta_2\Omega_2^{-1})^{(Q-(s+1))}(\Delta_2\Omega_1^{-1})^{((s+1)-i)} + \sum_{i=s+1}^{Q-1}(-1)^{Q-i}(\Delta_2\Omega_2^{-1})^{(Q-i)} + I + \sum_{i=Q+1}^{S}\left(\sum_{j=0}^{S-i}(-1)^{(2Q+1-i)}(\Delta_2\Omega_2^{-1})^{(S+s-(i+j+1))}(\Delta_2\Omega_1^{-1})^{(j+1)}(\Psi\Omega_2^{-1})\right)\Bigg]\mathbf{e} = 1.$$

4 System performance measures

The following system performance measures are derived to find the values of expected total cost.

4.1 Mean Inventory Level (M_I)

$$M_I = \sum_{j_1=1}^{S}\sum_{j_4=0}^{M} j_1\left[x^{(j_1,0,0,j_4)} + x^{(j_1,5,0,j_4)}\right] + \sum_{j_1=1}^{S}\sum_{j_2=1}^{4}\sum_{j_3=0}^{N}\sum_{j_4=0}^{M} j_1 x^{(j_1,j_2,j_3,j_4)}$$

4.2 Mean Reorder Rate (M_R)

$$M_R = \sum_{j_2=1}^{2}\sum_{j_3=0}^{N}\sum_{j_4=0}^{M}\mu_1 x^{(s+1,j_2,j_3,j_4)}$$

4.3 Mean Number of HPC in the Queue (M_{HP})

$$M_{HP} = \sum_{j_2=3}^{5}\sum_{j_3=1}^{N}\sum_{j_4=0}^{M} j_3\left[x^{(0,0,j_3,j_4)} + x^{(j_1,j_2,j_3,j_4)}\right] + \sum_{j_1=1}^{S}\sum_{j_2=1}^{4}\sum_{j_3=1}^{N}\sum_{j_4=0}^{M} j_3 x^{(j_1,j_2,j_3,j_4)}$$

4.4 Mean Number of LPC in the Orbit (M_{LPO})

$$\begin{aligned} M_{LPO} &= \sum_{j_2=3}^{5}\sum_{j_3=0}^{N}\sum_{j_4=1}^{M} j_4\left[x^{(0,0,j_3,j_4)} + x^{(j_1,j_2,j_3,j_4)}\right] + \sum_{j_1=1}^{S}\sum_{j_2=1}^{4}\sum_{j_3=0}^{N}\sum_{j_4=1}^{M} j_4 x^{(j_1,j_2,j_3,j_4)} \\ &+ \sum_{j_1=1}^{S}\sum_{j_4=1}^{M} j_4\left[x^{(j_1,0,0,j_4)} + x^{(j_1,5,0,j_4)}\right] \end{aligned}$$

4.5 Mean Interruption Rate (M_{IR})

$$M_{IR} = \sum_{j_3=0}^{N}\sum_{j_4=0}^{M} \eta x^{(0,3,j_3,j_4)} + \sum_{j_1=1}^{S}\sum_{j_3=0}^{N}\sum_{j_4=0}^{M} \eta\left[x^{(j_1,1,j_3,j_4)} + x^{(j_1,3,j_3,j_4)}\right]$$

4.6 Mean Repair Rate (M_{RR})

$$M_{RR} = \sum_{j_2=4}^{5}\sum_{j_3=0}^{N}\sum_{j_4=0}^{M} \alpha x^{(0,j_2,j_3,j_4)} + \sum_{j_1=1}^{S}\sum_{j_4=0}^{M} \alpha x^{(j_1,5,0,j_4)} + \sum_{j_1=1}^{S}\sum_{j_3=0}^{N}\sum_{j_4=0}^{M} \alpha\left[x^{(j_1,2,j_3,j_4)} + x^{(j_1,4,j_3,j_4)}\right]$$

4.7 Mean Number of HPC lost (M_{HPL})

$$M_{HPL} = \sum_{j_2=3}^{5}\sum_{j_4=0}^{M} \lambda_1\left[x^{(0,0,N,j_4)} + x^{(0,j_2,N,j_4)}\right] + \sum_{j_1=1}^{S}\sum_{j_2=1}^{4}\sum_{j_4=0}^{M} \lambda_1 x^{(j_1,j_2,N,j_4)}$$

4.8 Mean Number of LPC lost (M_{LPC})

$$M_{LPC} = \sum_{j_2=3}^{4}\sum_{j_3=0}^{N} \lambda_2 x^{(0,j_2,j_3,M)} + \sum_{j_1=1}^{S}\sum_{j_2=1}^{4}\sum_{j_3=0}^{N} \lambda_2 x^{(j_1,j_2,j_3,M)}$$

4.9 Expected total cost

Here different costs are defined as

c_h = The inventory carrying cost per unit item per unit time.
c_s = Setup cost per order.
c_i = Interruption rate per unit per unit time.
c_r = Repair rate per unit per unit time.
c_{wh} = Waiting time cost of a HP customer per unit per unit time.
c_{wl} = Waiting time cost of a LP customer per unit per unit time.
c_{lh} = Cost due to loss of HP customers per unit per unit time.
c_{ll} = Cost due to loss of LP customers per unit per unit time.

We introduce a cost function, defined as the expected total cost (TC) of the system, is given by

$$TC(S,s,N,M) = c_h\eta_I + c_s\eta_R + c_p\eta_P + c_{s1}E[W] + c_{s2}E[F] + c_{l1}\eta_{L1} + c_{l2}\eta_{L2}. \quad (3)$$

References

[1] Berman, O., and Kim, E. (1999). Stochastic inventory policies for inventory management of service facilities. Stochastic Models, 15, 695-718.

[2] Berman, O., and Sapna, K. P. (2000). Inventory management at service facilities for systems with arbitrarily distributed service times. Stochastic Models, 16, 343-360.

[3] Elango C, 2001, A continuous review perishable inventory system at service facilities, PhD dissertation,Madurai Kamaraj University, Madurai.

[4] Arivarignan, G., Elango, C. and Arumugam, N., (2002) A Continuous Review Perishable Inventory Control System at Service Facilities, Advances in Stochastic Modelling, in: J.R.Artalejo, A. Krishnamoorthy,(ED.).,Notable Publications, Inc., New Jersey, USA, pp.19 - 40.

[5] Arivarignan, G. and Sivakumar, B., (2003) Inventory System with Renewal Demands at Service Facilities, Stochastic Point Processes, in: S.K. Srinivasan and A. Vijayakumar (ED.), Narosa Publishing House, New Delhi, India, pp.108 - 123.

[6] Falin, G.I. and Templeton, J.G.C. (1997). Retrial Queues. Chapman and Hall, London.

[7] Artalejo J.R, Gomez-Corral A (2008) Retrial queueing systems: a computational approach. Springer,Berlin.

[8] Artalejo, J.R., Krishnamoorthy, A., Lopez-Herrero, M.J.: Numerical analysis of (s, S) inventorysystems with repeated attempts. Ann. Oper. Res. 141, 67-83 (2006).

[9] Ushakumari, P. V. (2006). On (s,S) inventory system with random lead time and repeated demands,Journal of Applied Mathematics and Stochastic Analysis, Vol.2006, 1-22.

[10] Choi B.D, Park KK (1990) The M/G/1 retrial queue with Bernoulli schedule. Queueing Syst 7(2):219-228.

[11] Krishnamoorthy A, Jose KP (2007) Comparison of inventory systems with service, positive leadtime,loss, and retrial of customers. J Appl Math Stoch Anal 2007, Article ID 37848, p 23. doi: 10.1155/2007/37848.

[12] A. Krishnamoorthy, M.E. Islam, (s, S) inventory system with postponed demands, Stochastic Analysis and Applications 22 (3) (2004) 827-842.

[13] B. Sivakumar, G. Arivarignan, An inventory system with postponed demands, Stochastic Analysis and Applications 22 (3) (2008) 827-842.

[14] Paul Manuel, B. Sivakumar, G. Arivarignan, Perishable inventory system with postponed demands and negative customers, Journal of Applied Mathematics and Decision Sciences 2007 (2007) 1-12.