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On ABC_4 and GA_5 Index of Subdivided and Line Graph of Subdivided Dutch Windmill Graph

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Abstract

Topological indices are the numeric descriptors of a graph which characterizes its topology and it is structure invariant. Topological indices introduced on the molecular structure can help material scientists to understand its chemical and biological features in better and easy way [2, 11, 13 14, 16]. In this paper, we present neighborhood based topological indices of sub-divided Dutch windmill graph and line graph of subdivided Dutch windmill graph.

Keywords: Atom bond connectivity index 4, Geometric Arithmetic index 5, Dutch windmill Graph D_3^n, Subdivided Graph, Line Graph.

1 Introduction

Topological indices are important tools for analyzing some physicochemical properties of molecules without performing any experiment [2, 3, 8, 9, 13, 14, 15, 24]. The Atom-Bond Connectivity index ABC [7, 17, 21] was defined by Ernesto Estrada [6] as follows

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}} \tag{1}$$

There are many open problems related to ABC index in the mathematical chemistry literature, interested reader can see [1, 5, 10, 22, 23]. Geometric-Arithmetic index introduced by Vukiceveic and Furtula [4, 18]. For a simple connected graph it is defined as follows

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v} \tag{2}$$

Recently, several authors have introduced new versions of the ABC and GA indices, which are mentioned below. The fourth version of ABC index is defined as [11]

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{s_u + s_v - 2}{s_u s_v}} \tag{3}$$

The fifth version of GA index is defined as [12]

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{s_u s_v}}{s_u + s_v} \tag{4}$$

The concept of topological indices came from Wiener [19], while he was working on the boiling points of paraffin[20]. Later, it was named after him as Wiener index.

2 Results and Discussion

In the field of graph theory, the Dutch windmill graph D_3^n also known as friendship graph, is obtained by taking n copies of cycle graph C_3 with a common vertex. Its definition can be extended to D_m^n by taking n copies of C_m, here we have worked on D_3^n. Let G_1 be the sub-divided Dutch windmill graph having $(4, 4)$, $(4, 2n + 2)$ and $(2n + 2, 4)$ neighborhood based types of edges each has count 2n. In this paper we have taken n=1 subdivision. Similarly, let G_2 be the line graph of sub-divided Dutch windmill graph it has $(4, 4)$, $(4, 2n + 2)$, $(2n + 2, 4n^2 - 2n + 2)$ and $(4n^2 - 2n + 2, 4n^2 - 2n + 2)$ types of edges with count n, 2n, 2n and $2n^2 - n$ respectively.

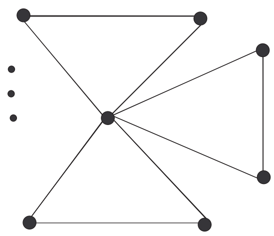

Figure 1: Dutch windmill graph

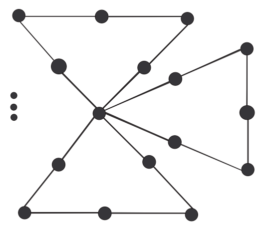

Figure 2: Subdivided Dutch windmill graph.

2.1 Theorem 1:

The ABC_4 and GA_5 index for sub-divided Dutch windmill graph are;

$$\begin{aligned} i).ABC_4(G_1) &= n\sqrt{\frac{3}{2}} + n\sqrt{\frac{n+2}{n+1}} + n\sqrt{\frac{3}{n+1}}, \\ ii).GA_5(G_1) &= 2n + \frac{4n\sqrt{2(n+1)}}{3+n} + \frac{4n\sqrt{2n(n+1)}}{3n+1}. \end{aligned}$$

Proof:
i). By using the information above and inseting the values in $equation(3)$, we get

$$\begin{aligned} ABC_4(G_1) &= 2n\sqrt{\frac{4+4-2}{(4)(4)}} + 2n\sqrt{\frac{4+2n+2-2}{(4)(2n+2)}} + 2n\sqrt{\frac{2n+2+4n-2}{(2n+2)(4n)}} \\ &= n\sqrt{\frac{3}{2}} + 2n\sqrt{\frac{2(n+2)}{8(n+1)}} + 2n\sqrt{\frac{6n}{8n(n+1)}} \\ &= n\sqrt{\frac{3}{2}} + \frac{2n}{2}\sqrt{\frac{n+2}{n+1}} + \frac{2n}{2}\sqrt{\frac{3}{n+1}} \\ &= n\sqrt{\frac{3}{2}} + n\sqrt{\frac{n+2}{n+1}} + n\sqrt{\frac{3}{n+1}} \end{aligned}$$

ii). By using the information above and inseting the values in $equation(4)$, we get

$$\begin{aligned} GA_5(G_1) &= \sum \frac{2\sqrt{(4)(4)}}{4+4} + \sum \frac{2\sqrt{(4)(2n+2)}}{4+2n+2} + \sum \frac{2\sqrt{(2n+2)(4n)}}{2n+2+4n} \\ &= \frac{(2n)(2)(4)}{8} + \frac{(2n)(2)\sqrt{(4)(2n+2)}}{4+2n+2} + \frac{(2n)(2)\sqrt{(2n+2)(4n)}}{2n+2+4n} \\ &= \frac{4n(4)}{8} + \frac{4n\sqrt{8(n+1)}}{6+2n} + \frac{4n\sqrt{8n(n+1)}}{6n+2} \\ &= 2n + \frac{4n\sqrt{2(n+1)}}{3+n} + \frac{4n\sqrt{2n(n+1)}}{3n+1} \end{aligned}$$

2.2 Theorem 2:

The ABC_4 and GA_5 index for line graph of sub-divided Dutch windmill graph are;

$$\begin{aligned} i).ABC_4(G_2) &= \frac{n}{2}\sqrt{\frac{3}{2}} + n\sqrt{\frac{n+2}{(n+1)}} + 2n\sqrt{\frac{2n^2+1}{(n+1)(4n^2-2n+2)}} \\ &\quad + \frac{2n^2-n}{4n^2-2n+2}\sqrt{8n^2-4n+2}, \\ ii).GA_5(G_2) &= \frac{4n\sqrt{2(n+1)}}{3+n} + \frac{2n\sqrt{(n+1)(2n^2-n+1)}}{n^2+1} + 2n^2. \end{aligned}$$

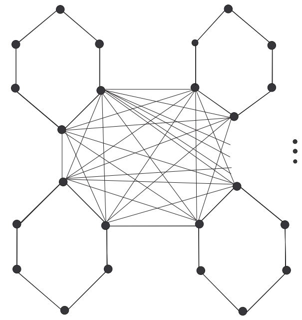

Figure 3: Line graph of subdivided Dutch windmill graph

Proof:
i). By using the information above and inseting the values in $equation(3)$, we get

$$\begin{aligned}
ABC_4(G_2) &= \sum_{uv\in E(G)}\sqrt{\frac{s_u+s_v-2}{s_us_v}}\\
&= \sum\sqrt{\frac{4+4-2}{(4)(4)}}+\sum\sqrt{\frac{4+2n+2-2}{(4)(2n+2)}}\\
&\quad+\sum\sqrt{\frac{2n+2+4n^2-2n+2-2}{(2n+2)(4n^2-2n+2)}}\\
&\quad+\sum\sqrt{\frac{4n^2-2n+2+4n^2-2n+2-2}{(4n^2-2n+2)(4n^2-2n+2)}}\\
&= n\sqrt{\frac{6}{16}}+2n\sqrt{\frac{4+2n}{4(2n+2)}}+2n\sqrt{\frac{2+4n^2}{(2n+2)(4n^2-2n+2)}}\\
&\quad+(2n^2-n)\sqrt{\frac{8n^2-4n+2}{(4n^2-2n+2)^2}}\\
&= \frac{n}{2}\sqrt{\frac{3}{2}}+2n\sqrt{\frac{2(2+n)}{4(2n+2)}}+2n\sqrt{\frac{4n^2+2}{(2n+2)(4n^2-2n+2)}}\\
&\quad+\frac{2n^2-n}{4n^2-2n+2}\sqrt{8n^2-4n+2}\\
&= \frac{n}{2}\sqrt{\frac{3}{2}}+n\sqrt{\frac{n+2}{(n+1)}}+2n\sqrt{\frac{2n^2+1}{(n+1)(4n^2-2n+2)}}\\
&\quad+\frac{2n^2-n}{4n^2-2n+2}\sqrt{8n^2-4n+2}
\end{aligned}$$

ii). By using the information above and inseting the values in $equation(4)$, we get

$$\begin{aligned}
GA_5(G_2) &= \sum_{uv\in E(G)} \frac{2\sqrt{s_u s_v}}{s_u+s_v} \\
&= \sum \frac{2\sqrt{(4)(4)}}{4+4} + \sum \frac{2\sqrt{(4)(2n+2)}}{4+2n+2} + \sum \frac{2\sqrt{(2n+2)(4n^2-2n+2)}}{2n+2+4n^2-2n+2} \\
&\quad + \sum \frac{2\sqrt{(4n^2-2n+2)(4n^2-2n+2)}}{4n^2-2n+2+4n^2-2n+2} \\
&= \frac{(n)(2)\sqrt{(4)(4)}}{4+4} + \frac{(2n)(2)\sqrt{(4)(2n+2)}}{4+2n+2} \\
&\quad + \frac{(2n)(2)\sqrt{(2n+2)(4n^2-2n+2)}}{2n+2+4n^2-2n+2} \\
&\quad + \frac{(2n^2-n)(2)\sqrt{(4n^2-2n+2)(4n^2-2n+2)}}{4n^2-2n+2+4n^2-2n+2} \\
&= \frac{(2)(n)(4)}{8} + \frac{4n\sqrt{8(n+1)}}{6+2n} + \frac{4n\sqrt{4(n+1)(2n^2-n+1)}}{4n^2+4} \\
&\quad + \frac{4(2n^2-n)(2n^2-n+1)}{8n^2-4n+4} \\
&= n + \frac{2n\sqrt{8(n+1)}}{3+n} + \frac{n\sqrt{4(n+1)(2n^2-n+1)}}{n^2+1} \\
&\quad + \frac{(2n^2-n)(2n^2-n+1)}{2n^2-n+1} \\
&= \frac{4n\sqrt{2(n+1)}}{3+n} + \frac{2n\sqrt{(n+1)(2n^2-n+1)}}{n^2+1} + 2n^2
\end{aligned}$$

3 Conclusion

In this paper by means of graph structure analysis, we have calculated the closed formulas for atom bond connectivity index version 4 and geometric arithmetic index version 5, for k=1 subdivided Dutch windmill graph and line graph of k=1 subdivided Dutch windmill graph. In future, some other structures can be studied. These calculated topological indices will help scientists to understand chemical and biological properties of compounds having such graphical structure.

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