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ESTIMATES FOR TWO-DIMENSIONAL OSCILLATORY INTEGRALS WITH SMOOTH PHASE

Gafurjan Khasanov[1] and Nodira Djurayeva[2]
[1]Samarkand state university , Samarkand, Uzbekistan
khasanov_g75@mail.ru
[2]Navoi State Mining Institute, Navoi, Uzbekistan

Abstract:In this paper we consider some two-dimensional oscillatory integrals associated integrals associated to Fourier transform of Borel's measures supported on two-dimensional smooth hypersurfaces in Euclidian space. We obtain uniform estimates for such integrals.

We consider both upper and lower estimates for oscillatory integrals. First, we define a class of amplitude functions. For this reason we consider a family of smooth curves $K = \{k = (x_1(\xi,\eta), x_2(\xi,\eta)) : (\xi,\eta) \in [-1;1] \times [-1;1]\}$, where (x_1, x_2) is a pair of fixed smooth functions [1,3].

The function $a \in A(K)$ if and only if there exists a fixed positive $C(a)$ such that for any $k \in K$ the following inequality:

$$\mathrm{V}[a \circ k] \leq C(a, K),$$

holds, where $\mathrm{V}[a \circ k]$is a total variation of the function $a \circ k$ on the interval $[-1;1]$. The class $A(K)$ is a normed spase with respect to norm

$$\|a\| = \sup_{\xi}(|a(x_1(\xi,-1), x_2(\xi,-1))| + \mathrm{V}[a \circ k]).$$

The class of amplitude functions A is defined by $\mathcal{A} = \cap\, A(K)$.

Let (r_1, r_2) be a pair of positive rational numbers. We define the norm in the space $C^\infty(\bar{U})$ by the following

$$\|f|\bar{U}\|_r = \max_{k_1 r_1 + k_2 r_2 \leq 1} \max_{x \in \bar{U}} \left|\frac{\partial^{|k|} f(x)}{\partial x_1^{k_1} \partial x_2^{k_2}}\right|.$$

Let $D = \{r^1, r^2, \ . \ . \ . \ , r^k\}$ be a finite number of pairs of positive rational numbers, U be a bounded neighborhood of the origin of R^2. We define a norm in the space $C^\infty(\bar{U})$ by the following

$$\|f|U\|_D = \max_{r^k \in D} \|f|U\|_{r^k}.$$

Let $f: (R^2, 0) \to (R, 0)$ be a smooth function in a neighborhood of the point (0,0).We assume the local coordinates system is fixed at the origin. We construct the Newton polygon $N(\tilde{f})$ in this coordinates system.

The union of all compact edges of Newton polygon is called to be a Newton diagram. It is denoted by $D(\tilde{f}) = \{\gamma_1, \gamma_2, . \ . \ . , \gamma_k\}$. We can define a pair of rational numbers $r_{\gamma\ell} = (r_{1\ell}, r_{2\ell})$ corresponding to the edge γ [1,2].

Let γ^p be the principal edge of the Newton polygon of f and (s, m) type of singularity of the polynomial P. Let F be a smooth function and $supp\ \tilde{F} \subset supp\ \tilde{f}$. Let $m = (m_1, m_2)$, $(M = (M_1, M_2))$ be point corresponding to the least (gratest) compact edge of Newton polygon of F. We suppose that the function F can be written as

$$F(x_1, x_2) = x_1^{m_1} x_2^{M_2} F_1(x_1, x_2),$$

whereF_1 is a smooth function.

If $supp\ \tilde{F} \subset supp\ \tilde{f}$ and F satisfies the above-mentioned condition, the we will write $supp\ \tilde{F} \subset\subset supp\ \tilde{f}$.

If $supp\ \tilde{F} \cap D \neq \emptyset$, then the function can be written as

$$F(x_1, x_2) = F_{\gamma_1}(x_1, x_2) + \tilde{F}(x_1, x_2),$$

where$F_{\gamma_1}(x_1, x_2)$ is a weighted homogenous part of the function F and $\tilde{F} \in I_{r_1}$. We can write the function $\tilde{F}(x_1, x_2)$ in the form

$$\tilde{F}(x_1, x_2) = \sum_{r_{11} i + r_{12} j = 1} x_1^i x_2^j b_{ij}(x_1, x_2)$$

where$b_{ij}(x_1, x_2) \in \mathfrak{M}$, $b_{ij}(0,0) = 0$. $\mathfrak{M}$ is the maximal ideal of ring of germs of smooth functions at the origin.

Lemma. For any positive number ε there exists a positive number $\delta > 0$ such that for any $|x_1| < \delta$, $|x_2| < \delta$ and for any $r_{11} i + r_{12} j = 1$ the inequality

$$b_{ij}(x_1, x_2) < \varepsilon$$

holds.

The following theorem shows that the oscillation exponent does not change under small perturbation of the Newton polygon [4-5].

Theorem. Let $f:(R^2,0)\to(R,0)$ be a smooth function in a neighborhood of the point $(0,0)$. Then there exist a neighborhood U of the origin and positive numbers ε, C such that for any function F satisfying the following conditions:

1) $supp\ \tilde{F} \subset\subset N(\tilde{f})$
2) $\|f|U\|_{D(f)} < \varepsilon$
3) $a \in \mathcal{A}(U)$

then the following inequality

$$\left|\int_{R^2} a(x)exp(it(f+F)(x))dx\right| \leq C\|a\|_{V}|t|^{-s}|lnt|^m$$

holds, where (s,m) is the type of the principal part of the function f at the origin.

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SHORT BIODATA OF ALL THE AUTHOR

Gafurjan KHASANOV
He is graduated from the Samarkand State University. His scientific interests are connected with the damped oscillator integrals and boundedness of maximal operators. At the present Dr. G.Khasanov is a PostDoc at the Samarkand State University.

Nodira DJURAEVA
She is graduated from the Samarkand State University. Her scientific interests are connected with the damped oscillator integrals and boundedness of maximal operators. At the present N.Djuraeva is a senior teacher of the Department of Higher mathematics and Information Technologies at the Navoi State Mining Institute